(2) $P_4(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4$

(4) $P_5(x) = e - ex + \frac{e}{2!}x^2 - \frac{e}{3!}x^3 + \frac{e}{4!}x^4 - \frac{e}{5!}x^5$

$P(x) = e - ex + \frac{e}{2!}x^2 - \dots (-1)^n \frac{e\,x^n}{n!}$

p 491

Maclaurin Series

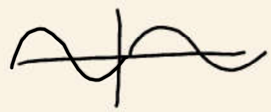
$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n \quad (|x| < 1)$$

$$\frac{1}{1+x} = 1 - x + x^2 - \cdots + (-x)^n + \cdots = \sum_{n=0}^{\infty} (-1)^n x^n \quad (|x| < 1)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{(all real } x\text{)}$$

p491

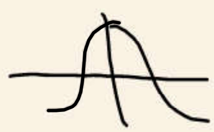
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots$$



$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \quad \text{(all real } x\text{)}$$

HW: p492 #1-4, 23

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots$$



$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \quad \text{(all real } x\text{)}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + \cdots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \quad (-1 < x \le 1)$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \quad (|x| \le 1)$$

Find a series for: (centered at $x=0$)

① $\cos 2x = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \cdots$

$= 1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \cdots$

② $\frac{1+\cos 2x}{2} = \frac{1 + \left(1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \cdots\right)}{2}$

$= \frac{2 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \cdots}{2}$

$= 1 - x^2 + \frac{1}{3}x^4 - \frac{2}{45}x^6 + \cdots$

③ $x \sin x = x\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right)$

$= x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!} + \cdots$

④ e^{1-x}

$e^1 \cdot e^{-x}$

$e \cdot e^{-x}$

$e \cdot \left(1 + (-x) + \frac{(-x)^2}{2!} + \frac{(-x)^3}{3!} + \cdots\right)$

$e \cdot \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right)$

$= e - ex + e\frac{x^2}{2!} - e\frac{x^3}{3!} + \cdots$

Taylor Series generated by f at $x=0$
(aka MacLaurin Series)

$f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + \frac{f'''(0)}{3!}x^3 + \cdots$

Taylor Series centered at $x=a$:

$\boxed{f(a) + f'(a)(x-a)} + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$

$L(x)$!

HW: p492
#5-9, 17, 18, 24, 25ab, 32, 33
and 2 FRQ's